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Dirac equation in the confining SU(3)-Yang-Mills field and relativistic effects in charmonium spectrum

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Abstract

The recently obtained solutions of the Dirac equation in the confining SU(3)-Yang-Mills field in Minkowski spacetime are applied to describe the energy spectrum of charmonium. The nonrelativistic limit is considered for the relativistic effects to be estimated in a self-consistent way and it is shown that the given effects could be extremely important for both the energy spectrum and the confinement mechanism.

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1 Introduction

Theory of quarkonium ranks high within hadron physics as the one of central sources of information about the quark interaction. Referring for more details to the recent up-to-date review[1], it should be noted here that at present some generally accepted relativistic model of quarkonium is absent. The description of quarkonium is actually implemented by nonrelativistic manner on the basis of the Schrödinger equation (concerning the general ideology here see, *e. g.*, ref. [2]) and then one tries to include relativistic corrections in one or another way. Such an inclusion is not single-valued and varies in dependence of the point of view for different authors (see, *e. g.*, ref.[3] and references therein). It would be more consistent, to our mind, building a primordially relativistic model so that one can then pass on to the nonrelativistic one by the standard limiting transition and, thus, to estimate the relativistic effects in a self-consistent way. As follows from the main principles of quantum chromodynamics (QCD), the suitable relativistic models for a description of the relativistic bound states of quarkonium could consist in considering the solutions of the Dirac equation in a SU(3)-Yang-Mills field representing the gluonic field. Indeed, the Dirac equation in a SU(3)-Yang-Mills field is the direct consequence of the QCD Lagrangian in the same way as the Dirac equation for the hydrogen atom is the direct consequence of the quantum electrodynamics (QED) Lagrangian. Following the latter analogy, the mentioned SU(3)-Yang-Mills field should be the so-called confining solution of the corresponding Yang-Mills equations, *i. e.*, it should model the quark confinement. Such solutions are usually supposed to contain at least one component of the mentioned SU(3)-field linear in r, the distance between quarks. Recently, in Ref.[4] a number of such solutions have been obtained and the corresponding spectrum of the Dirac equation describing the relativistic bound states in those confining SU(3)-Yang-Mills fields has been analysed. In this paper we should like to apply the results of ref. [4] to the description of the charmonium spectrum. Here we solve the inverse problem, *i. e.*, we define the confining gluonic field components in the covariant description (SU(3)-connection) for charmonium (which corresponds to a potential of $q\overline{q}$-interaction in the nonrelativistic description) employing the experimental data on the mentioned spectrum[5]. As a consequence, we shall not use any nonrelativistic potentials modelling confinement, for example, of the harmonic oscillator or funnel types, in particular, because the latter do not satisfy the Yang-Mills equations, while the SU(3)-gluonic field used by us does. Accordingly, in our case the approach is relativistic from the very outset and our considerations are essentially nonperturbative, since we shall not use any expansions in the coupling constant g or in any other parameters.

Further we shall deal with the metric of the flat Minkowski spacetime M that we write down (using the ordinary set of local spherical coordinates r, ϑ, φ for the spatial part) in the form

$$ds^2 = g_{\mu\nu} dx^\mu \otimes dx^\nu \equiv dt^2 - dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \, . \tag{1}$$

Besides, we have $|\delta| = |\det(g_{\mu\nu})| = (r^2 \sin\vartheta)^2$ and $0 \le r < \infty$, $0 \le \vartheta < \pi$, $0 \le \varphi < 2\pi$.

Throughout the paper we employ the system of units with $\hbar = c = 1$, unless explicitly stated otherwise. Finally, we shall denote $L_2(F)$ the set of the modulo

square integrable complex functions on any manifold F furnished with an integration measure while $L_2^n(F)$ will be the n-fold direct product of $L_2(F)$ endowed with the obvious scalar product.

2 Preliminaries

To formulate the results of ref.[4] that we need here, let us notice that the relativistic wave function of the quarkonium can be chosen in the form $\psi = (\psi_1, \psi_2, psi_3)$ with the four-dimensional spinors ψ_j representing the j-th colour component of the quarkonium. The corresponding Dirac equation for ψ may look as follows:

$$\mathcal{D}\psi = \mu_0\psi, \tag{2}$$

where μ_0 is a mass parameter and one can consider it to be the reduced relativistic mass which is equal, *e. g.* for quarkonia, to one half the current mass of quarks forming a quarkonium, while the coordinate r stands for the distance between quarks.

From general considerations the explicit form of the operator $\mathcal{D}$ in local coordinates x^μ on Minkowski manifold can be written as follows:

$$\mathcal{D} = i(\gamma^e \otimes I_3)E_e^\mu(\partial_\mu \otimes I_3 - \frac{1}{2}\omega_{\mu ab}\gamma^a\gamma^b \otimes I_3 - igA_\mu),\ a < b, \tag{3}$$

where $A = A_\mu dx^\mu$, $A_\mu = A^c_\mu T_c$ is a SU(3)-connection in the (trivial) three-dimensional bundle ξ over the Minkowski spacetime, I_3 is the unit matrix 3×3, the matrices T_c form a basis of the Lie algebra of SU(3) in the 3-dimensional space (we consider T_a to be Hermitian which is acceptable in physics), $c = 1, ..., 8$, $\otimes$ here means tensorial product of matrices, g is a gauge coupling constant. At last the coefficients E_e^μ and $\omega_{\mu ab}$ depend on the choice of metric and their form for metric (1) can be found in ref. [4] as well as the explicit presentation of matrices γ^a, $a = 0, ..., 3$. As for the connection A_μ in bundle ξ, then, the suitable one should be the confining solution of the Yang-Mills equations

$$d * F = *F \wedge A - A \wedge *F \tag{4}$$

with the exterior differential $d = \partial_t dt + \partial_r dr + \partial_\vartheta d\vartheta + \partial_\varphi d\varphi$ in coordinates t, r, ϑ, φ while the curvature matrix (field strentgh) for ξ-bundle is $F = dA + A \wedge A$ and $*$ means the Hodge star operator conforming to metric (1).

In ref.[4] the black hole physics techniques from refs.[6] were used to find a set of the confining solutions of eq. (4). For the aims of the given paper we need one of these solutions of ref.[4]. Let us adduce it here putting $T_c = \lambda_c$, where λ_c are the Gell-Mann matrices (whose explicit form can be found in refs.[6]). Then the solution in question is the following one:

$$A_t^3 + \frac{1}{\sqrt{3}}A_t^8 = -\frac{a_1}{r} + A_1, -A_t^3 + \frac{1}{\sqrt{3}}A_t^8 = \frac{a_1 + a_2}{r} - (A_1 + A_2), -\frac{2}{\sqrt{3}}A_t^8 = -\frac{a_2}{r} + A_2,$$

$$A_\varphi^3 + \frac{1}{\sqrt{3}}A_\varphi^8 = b_1 r + B_1, -A_\varphi^3 + \frac{1}{\sqrt{3}}A_\varphi^8 = -(b_1 + b_2)r - (B_1 + B_2), -\frac{2}{\sqrt{3}}A_\varphi^8 = b_2 r + B_2 \tag{5}$$

with all other $A^c_\mu = 0$, where real constants a_j, A_j, b_j, B_j parametrize the solution, and we wrote down the solution in the combinations that are just needed

to insert into (2). From the adduced form it is clear that the solution is a configuration describing the electric Coulomb-like colour field (components A_t) and the magnetic colour field linear in r (components A_φ). Also, it is easy to check that the given solution satisfy the Lorentz gauge condition that can be written in the form $\mathrm{div}(A) = 0$, where the divergence of the Lie algebra valued 1-form $A = A^c_\mu T_c dx^\mu$ is defined by the relation

$$\mathrm{div}(A) = \frac{1}{\sqrt{|\delta|}} \partial_\mu (\sqrt{|\delta|} g^{\mu\nu} A_\nu) \,. \tag{6}$$

As was shown in ref.[4], after inserting the above confining solution into Eq. (2), the latter admits the solutions of the form

$$\psi_j = e^{i\omega_j t} r^{-1} \begin{pmatrix} F_{j1}(r)\Phi_j(\vartheta,\varphi) \\ F_{j2}(r)\sigma_1\Phi_j(\vartheta,\varphi) \end{pmatrix} , j = 1,2,3 \tag{7}$$

with the 2D eigenspinor $\Phi_j = \begin{pmatrix} \Phi_{j1} \\ \Phi_{j2} \end{pmatrix}$ of the Euclidean Dirac operator on the unit sphere $\mathbb{S}^2$. The explicit form of Φ_j is not needed here and can be found in refs.[7]. For the purpose of the present paper it is sufficient to know that spinors Φ_j can be subject to the normalization condition

$$\int_0^\pi \int_0^{2\pi} (|\Phi_{j1}|^2 + |\Phi_{j2}|^2) \sin\vartheta d\vartheta d\varphi = 1 \,, \tag{8}$$

i. e., they form an orthonormal basis in $L_2^2(\mathbb{S}^2)$.
The energy spectrum ε of a quarkonium is given (in a more symmetrical form than in ref. [4]) by the relation $\varepsilon = \omega_1 + \omega_2 + \omega_3$ with

$$\omega_1 = \omega_1(n_1, l_1, \lambda_1) = \frac{-\Lambda_1 g^2 a_1 b_1 + (n_1 + \alpha_1)\sqrt{(n_1^2 + 2n_1\alpha_1 + \Lambda_1^2)\mu_0^2 + g^2 b_1^2 (n_1^2 + 2n_1\alpha_1)}}{n_1^2 + 2n_1\alpha_1 + \Lambda_1^2}, \tag{9}$$

$$\omega_2 = \omega_2(n_2, l_2, \lambda_2) =$$
$$\frac{-\Lambda_2 g^2 (a_1 + a_2)(b_1 + b_2) - (n_2 + \alpha_2)\sqrt{(n_2^2 + 2n_2\alpha_2 + \Lambda_2^2)\mu_0^2 + g^2 (b_1 + b_2)^2 (n_2^2 + 2n_2\alpha_2)}}{n_2^2 + 2n_2\alpha_2 + \Lambda_2^2}, \tag{10}$$

$$\omega_3 = \omega_3(n_3, l_3, \lambda_3) = \frac{-\Lambda_3 g^2 a_2 b_2 + (n_3 + \alpha_3)\sqrt{(n_3^2 + 2n_3\alpha_3 + \Lambda_3^2)\mu_0^2 + g^2 b_2^2 (n_3^2 + 2n_3\alpha_3)}}{n_3^2 + 2n_3\alpha_3 + \Lambda_3^2}, \tag{11}$$

where $\Lambda_1 = \lambda_1 - gB_1$, $\Lambda_2 = \lambda_2 + g(B_1 + B_2)$, $\Lambda_3 = \lambda_3 - gB_2$, $n_j = 0, 1, 2, ...$, while $\lambda_j = \pm(l_j + 1)$ are the eigenvalues of euclidean Dirac operator on unit sphere with $l_j = 0, 1, 2, ...$ Besides,

$$\alpha_1 = \sqrt{\Lambda_1^2 - g^2 a_1^2} \,, \alpha_2 = \sqrt{\Lambda_2^2 - g^2 (a_1 + a_2)^2} \,, \alpha_3 = \sqrt{\Lambda_3^2 - g^2 a_2^2} \,. \tag{12}$$

Further, the radial part of (7), for instance, for the ψ_1-component, is given at $n_1 = 0$ by

$$F_{11} = C_1 A r^{\alpha_1} e^{-\beta_1 r} \left(1 - \frac{Y_1}{Z_1}\right), F_{12} = i C_1 B r^{\alpha_1} e^{-\beta_1 r} \left(1 + \frac{Y_1}{Z_1}\right), \tag{13}$$

while at $n_1 > 0$ by

$$F_{11} = C_1 A r^{\alpha_1} e^{-\beta_1 r}\left[\left(1-\frac{Y_1}{Z_1}\right)L^{2\alpha_1}_{n_1}(r_1)+\frac{AB}{Z_1}r_1 L^{2\alpha_1+1}_{n_1-1}(r_1)\right],$$

$$F_{12} = iC_1 B r^{\alpha_1} e^{-\beta_1 r}\left[\left(1+\frac{Y_1}{Z_1}\right)L^{2\alpha_1}_{n_1}(r_1)-\frac{AB}{Z_1}r_1 L^{2\alpha_1+1}_{n_1-1}(r_1)\right], \qquad (14)$$

with the Laguerre polynomials $L^{\rho}_{n_1}(r_1)$, $r_1 = 2\beta_1 r$, $\beta_1 = \sqrt{\mu_0^2-(\omega_1-gA_1)^2+g^2b_1^2}$, $A = gb_1+\beta_1$, $B = \mu_0+\omega_1-gA_1$, $Y_1 = [\alpha_1\beta_1-ga_1(\omega_1-gA_1)+g\alpha_1 b_1]B+g^2a_1b_1A$, $Z_1 = [(\lambda_1-gB_1)A+ga_1\mu_0)]B+g^2a_1b_1A$. Finally, C_1 is determined from the normalization condition

$$\int_0^\infty (|F_{11}|^2+|F_{12}|^2)dr = \frac{1}{3}\,. \qquad (15)$$

Analogous relations will hold true for $\psi_{2,3}$, respectively, by replacing $a_1, A_1, b_1, B_1, \alpha_1 \to a_2, A_2, b_2, B_2, \alpha_3$ for ψ_3 and $a_1, A_1, b_1, B_1, \alpha_1 \to -(a_1+a_2), -(A_1+A_2), -(b_1+b_2), -(B_1+B_2), \alpha_2$ for ψ_2 so that $\beta_2 = \sqrt{\mu_0^2-[\omega_2+g(A_1+A_2)]^2+g^2(b_1+b_2)^2}$, $\beta_3 = \sqrt{\mu_0^2-(\omega_3-gA_2)^2+g^2b_2^2}$. Consequently, we shall gain that $\psi_j \in L_2^4(\mathbb{R}^3)$ at any $t \in \mathbb{R}$ and, as a result, the solutions of (7) may describe relativistic bound states of a quarkonium with the energy spectrum (9)–(11).

Before apllying the above relations to a description of charmonium spectrum let us adduce the nonrelativistic limits (*i. e.*, at $c \to \infty$) for the energies of (9)–(11). The common case is not needed to us in the present paper, so we shall restrict ourselves to the case of $n_j = 0, 1$ and $l_j = 0$. Expanding ω_j in $x = \frac{g}{\hbar c}$, we get

$$\omega_1(0,0,\lambda_1) = -x\frac{ga_1b_1}{\lambda_1}+\mu_0c^2\left[1-\frac{1}{2}\left(\frac{a_1}{\lambda_1}\right)^2x^2+O(x^3)\right],$$

$$\omega_1(1,0,\lambda_1) = -x\frac{ga_1b_1}{4\lambda_1}+\mu_0c^2\left[1-\frac{1}{8}\left(\frac{a_1}{\lambda_1}\right)^2x^2+O(x^3)\right], \qquad (16)$$

which yields at $c \to \infty$ (putting $\hbar = c = 1$ again)

$$\omega_1(0,0,\lambda_1) = \mu_0\left[1-\frac{1}{2}\left(\frac{ga_1}{\lambda_1}\right)^2\right], \omega_1(1,0,\lambda_1) = \mu_0\left[1-\frac{1}{8}\left(\frac{ga_1}{\lambda_1}\right)^2\right]. \qquad (17)$$

Analogously, we shall have

$$\omega_2(0,0,\lambda_2) = -\mu_0\left[1-\frac{1}{2}\left(\frac{g(a_1+a_2)}{\lambda_2}\right)^2\right],$$

$$\omega_2(1,0,\lambda_2) = -\mu_0\left[1-\frac{1}{8}\left(\frac{g(a_1+a_2)}{\lambda_2}\right)^2\right], \qquad (18)$$

$$\omega_3(0,0,\lambda_3) = \mu_0\left[1-\frac{1}{2}\left(\frac{ga_2}{\lambda_3}\right)^2\right], \omega_3(1,0,\lambda_3) = \mu_0\left[1-\frac{1}{8}\left(\frac{ga_2}{\lambda_3}\right)^2\right], \qquad (19)$$

where, of course, $\lambda_j = \pm 1$ and $\lambda_j^2 = 1$.

Table 1: Gauge coupling constant, mass parameter μ_0 and parameters of the confining SU(3)-connection for charmonium.

g	μ_0 (GeV)	a_1	a_2	b_1 (GeV)	b_2 (GeV)	B_1	B_2
4.68010	0.627818	0.0516520	2.45565	-0.705320	1.70660	2.10247	4.25862

3 Relativistic spectrum of charmonium

Now we can adduce numerical results for constants parametrizing the charmonium spectrum which are shown in table I.
One can note that the obtained mass parameter μ_0 is consistent with the present-day experimental limits [5] where the current mass of c-quark ($2\mu_0$) is accepted between 1.1 GeV and 1.4 GeV. As for parameters $A_{1,2}$ of solution (5), only the wave functions depend on them while the spectrum does not and within the present paper we consider $A_1 = A_2 = 0$.
With the constants of Table I, the present-day levels of the charmonium spectrum were calculated with the help of (9)–(11) while their nonrelativistic values with the aid of (17)–(19), according to the following combinations (we use the notations of levels from ref.[5]):

$$
\begin{aligned}
\eta_c(1S) &: \varepsilon_1 = \omega_1(0,0,-1) + \omega_2(0,0,-1) + \omega_3(0,0,-1)\,, \\
J/\psi(1S) &: \varepsilon_2 = \omega_1(0,0,-1) + \omega_2(0,0,1) + \omega_3(0,0,-1)\,, \\
\chi_{c0}(1P) &: \varepsilon_3 = \omega_1(0,0,-1) + \omega_2(0,0,-1) + \omega_3(0,0,1)\,, \\
\chi_{c1}(1P) &: \varepsilon_4 = \omega_1(0,0,1) + \omega_2(0,0,1) + \omega_3(0,0,1)\,, \\
\eta_c(1P) &: \varepsilon_5 = \omega_1(0,0,1) + \omega_2(1,0,-1) + \omega_3(1,0,-1)\,, \\
\chi_{c2}(1P) &: \varepsilon_6 = \omega_1(0,0,-1) + \omega_2(1,0,-1) + \omega_3(1,0,-1)\,, \\
\eta_c(2S) &: \varepsilon_7 = \omega_1(0,0,1) + \omega_2(1,0,1) + \omega_3(1,0,-1)\,, \\
\psi(2S) &: \varepsilon_8 = \omega_1(0,0,-1) + \omega_2(1,0,1) + \omega_3(1,0,-1)\,, \\
\psi(3770) &: \varepsilon_9 = \omega_1(1,0,-1) + \omega_2(1,0,-1) + \omega_3(0,0,1)\,, \\
\psi(4040) &: \varepsilon_{10} = \omega_1(0,0,1) + \omega_2(0,0,-1) + \omega_3(1,0,-1)\,, \\
\psi(4160) &: \varepsilon_{11} = \omega_1(0,0,1) + \omega_2(0,0,1) + \omega_3(1,0,-1)\,, \\
\psi(4415) &: \varepsilon_{12} = \omega_1(0,0,1) + \omega_2(0,0,-1) + \omega_3(1,0,1)\,. \qquad (20)
\end{aligned}
$$

Table II contains experimental values of these levels (from ref.[5]) and our theoretical relativistic and nonrelativistic ones, and also the contribution of relativistic effects in %, where it makes sense to speak about such a contribution. Besides, one can notice that the form of the wave functions (13)–(14) permits to consider, for instance, the quantity $1/\beta_1$ to be a characteristic size of quarkonium

Table 2: Experimental and theoretical charmonium levels.

ε_j	Experim. (GeV)	Relativ. (GeV)	Nonrelativ. GeV (GeV)	Relativ. contrib. (%)	r (fm)	r_0 (fm)	r_0/r
ε_1	2.97980	2.97980	2.37202	20.3965	0.0603473	1.32755	21.9984
ε_2	3.09688	3.09687	2.37202	23.4060	0.0603473	1.32755	21.9984
ε_3	3.41730	3.41729	2.37202	30.5877	0.0603473	1.32755	21.9984
ε_4	3.51053	3.51764	2.37202	32.5678	0.0602972	1.32755	22.0167
ε_5	3.52614	3.53085	1.05011	70.2590	0.0602972	1.32755	22.0167
ε_6	3.55617	3.54759	1.05011	70.3993	0.0603473	1.32755	21.9984
ε_7	3.59400	3.65282	1.05011	71.2521	0.0602972	1.32755	22.0167
ε_8	3.68600	3.66955	1.05011	71.3831	0.0603473	1.32755	21.9984
ε_9	3.76990	3.75160	-30.0324		0.0655399	2.64046	40.2878
ε_{10}	4.04000	4.04906	33.4683		0.0602972	1.32755	22.0167
ε_{11}	4.16000	4.16614	33.4683		0.0602972	1.32755	22.0167
ε_{12}	4.41500	4.41872	33.4683		0.0602972	1.32755	22.0167

state. Under the circumstances, if one calculates $1/\beta_1$ in both the relativistic ($b_1 \neq 0$) and nonrelativistic ($b_1 = 0$) cases, then one can obtain those sizes r and r_0 in fm (1 fm = 10^{-13} cm) so the latter are adduced in table II together with the quantity r/r_0.

4 Physical interpretation

The results obtained allow us to draw a number of conclusions. As is seen from table II, relativistic values are in good agreement with experimental ones, while nonrelativistic ones are not. The contribution of relativistic effects can amount to tens per cent and they cannot be considered as small, as was expected by a number of theorists [3]. Moreover, the more excited the state of charmonium the worse the nonrelativistic approximation. For very excited states, the latter is not applicable at all. The physical reason of it is quite clear. Really, we have seen in the nonrelativistic limit (see the relations (16)–(19)) that the parameters $b_{1,2}, B_{1,2}$ (see eq. (5)) of the linear interaction between quarks vanish under this limit and the nonrelativistic spectrum is independent of them and is practically getting the pure Coulomb one. As a consequence, the picture of linear confinement for quarks should be considered as an essentially relativistic one while the nonrelativistic limit is only a rather crude approximation. In fact, as follows from exact solutions of SU(3)-Yang–Mills equations of (5), the linear interaction between quarks is connected with colour magnetic field that dies out in the nonrelativistic limit, *i.e.* for static quarks. Only for the moving rapidly enough quarks the above field will appear and generate linear confinement between them. So the spectrum will depend on both the static Coulomb colour electric field and the dynamical colour magnetic field responsible for the

linear confinement for quarks which is just confirmed by our considerations. In our case, the interaction effect with the colour magnetic field is taken into consideration from the very outset just reflects the linear confinement at large distances.
Also, one can notice from table II that $r_0/r >> 1$ for all the charmonium states, which additionally points out the importance of the relativistic effects connected with colour magnetic field for confinement.
Finally, I would like to say a few words concerning the nonrelativistic potential models often used in quarkonium theory. The potentials between quarks here are usually modelled by those of harmonic oscillator or of funnel type (*i. e.*, of the form $\alpha/r + \beta r$ with some constants α and β), see, *e. g.*, refs. [8, 9].
It is clear, however, that from the QCD point of view the interaction between quarks should be described by the whole SU(3)-connection $A_\mu = A_\mu^c T_c$, genuinely relativistic object, the nonrelativistic potential being only some component of A_t^c surviving in the nonrelativistic limit at $c \to \infty$. As is easy to show, however, the connection of form $A_t^c = Br^\gamma$, where B is a constant, may be solution of the Yang-Mills equations (4) only at $\gamma = -1$, *i. e.* in the Coulomb-like case. Consequently, the potentials employed in nonrelativistic approaches do not obey the Yang-Mills equations. The latter ones are essentially relativistic and, as we have seen, the components linear in r of the whole A_μ are different from A_t and related with colour magnetic field vanishing in the nonrelativistic limit. That is why the nonrelativistic potential approach seems to be inconsistent though it developed a number of techniques and physical interpretations (in particular in charmonium theory [8, 9] which can be useful under a relativistic description as well. Our approach uses only the exact solutions of Yang-Mills equations as well as in atomic physics the interaction among particles (*e. g.*, the electric Coulomb one) is always the exact solution of the Maxwell equations (the particular case of the Yang-Mills equations).

5 Concluding Remarks

As we have seen, the application of the Dirac equation to the charmonium specrtum leads to a reasonable physical picture. From the obtained results there follows that the standard approach of potential models on the base of the Schrödinger Equationwith some potential modelling confinement seems to be inconsistent. The more consistent approach could be o the basis of the Schrödinger Equation in (colour) magnetic field since the linear confinement at large distances could be connected with the colour magnetic field rather than with the static colour electric one which follows from the exact solution of the SU(3)-Yang-Mills equations (see ref. [4] and the solution (5)). Historically, the latter way was rejected due to incomprehensible reasons. The analysis of the present paper shows, however, that the most consistent approach is probably the one based on the Dirac equation in the confining SU(3)-Yang-Mills field when the theory is relativistic from the very outset. In its turn, this approach is the direct consequence of the *relativistic* QCD Lagrangian since the mentioned Dirac equation is derived just from the latter one.
The calculations of the present paper can be extended. Indeed we have the explicit form (7) for the relativistic wave functions of a quarkonium that may be applied to the analysis of the quarkonium radiative decays. Besides, our

preliminary calculations show similar results to hold true also for bottomonium. At last, there is a possibility of modifying the gluon propagator on the basis of exact solutions of the SU(3)-Yang-Mills equations described here and in ref. [4] for the mentioned propagator to be able to lead to linear confinement between quarks at large ditances. The author hopes to discuss the mentioned questions elsewhere.

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